4. Shear Stress on the Water Side

The interfacial shear stress τ is caused by the drag of water upon grains protruding from the cutting surface. To construct a model of τ, one must understand the general nature of the flow around a grain. The typical grain will be treated as a rough sphere, having a diameter g small compared with the stream depth d so that the grain participates in the hydrodynamic environment of the cutting surface. Consider, as a specific instance, the Olsen and Thomas experiment described in Section 7. P_o was 17,000 psi, and therefore u_o = 1600 ft/sec according to the Bernoulli equation (6). The grain diameter g was about 0.005 inch, reasonably small compared with the 0.030-inch diameter of the nozzle.

The speed a of sound in water is 4800 ft/sec, so the Mach number u/a of the flow around a grain was modest,

$$\frac{u}{a} \approx \frac{1600}{4800} \approx 0.3 \ ,$$

small enough that the flow could be considered incompressible if it were steady. Grain removal could result in nonsteady compressibility effects, called "water hammer" in hydraulics, and water hammer has been advanced as a mechanism for hydraulic rock cutting [7]. But water hammer would be important only if a typical grain-removal time t were so short that the Helmholtz number g/at were of order unity. It is hard to imagine t being shorter than g/u, the time required for a grain to traverse its own diameter while moving with the flow, so the Helmholtz and Mach numbers are of the same order u/a. Neither steady nor non-steady compressibility effects should be important. Likewise with viscous effects, because the Reynolds number of the flow around the grain is high,

$$\frac{\rho u g}{\eta} \approx 58{,}000 \ ,$$

where η is the viscosity of the water.

The essential phenomenon governing the shear stress is cavitation. The cavitation number at the cutting surface is

$$\frac{p_s - p_v}{\frac{1}{2}\rho\, u^2} \ ,$$

where p_v is the vapor pressure of the water, and ρu^2 can be taken as the average